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STUDY OF THE DISTORTION OF PLATES OBTAINED WITH THE ELECTRONIC CAMERA

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STUDY OF THE DISTORTION OF PLATES OBTAINED WITH THE ELECTRONIC CAMERA

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ABSTRACT. The radial velocity of Mercury using an optical method was measured to derive the astronomical unit in meters. Optical characteristics of the optics used for the reduction were determined using special plates.

We carried out measurements of the radial velocity of Mercury using an optical method in order to derive a value of the astronomical unit in meters [1]. The difficulties in observing this heavenly body led us to use an electronic camera as the receiving device. For the measurements which follow and which are in progress at the present time, we attached a special spectrograph to the 1.93 meter telescope of the Haute-Provence Observatory. This apparatus will be described in a later publication.

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Since this method requires a considerable amount of effort, we first tried the techniques and the methods using a simple configuration, which was built at the Paris Observatory in 1961. We also observed a brighter heavenly body which is more easily observable, but of the same type: Venus [2].

Among the various problems which we had to study, we emphasize the distortion introduced by the electronic optics. In our case, it was necessary to correct the positions of the measured points on the

*Paris Observatory

**Numbers in the margin indicate the pagination in the original foreign text.

electronographic plate in order to place them at their "undistorted" positions with an error on the order of one micron. This could be done in two ways:

a - measure the distortion on each plate,

b - or, determine the characteristics of the optics used with special plates. This is done in order to insure that, for the required approximation, the distortion is independent of the small variations of the values of the voltages applied to the electrodes of the camera and independent of the mechanical modifications which can occur when its optics are disassembled.

Method a, which we used for reducing the plates obtained in 1961, seemed unsatisfactory because it gave no information regarding the possible anisotropy of the distortion. Also, in the present measurements, the two methods a and b were used concurrently.

It should also be noted that the electronic optics used for the measurements of Venus in 1961 and the optics used at the present since 1962 for measurements of Mercury do not have the same characteristics. We will call them optics I and optics II, respectively.

Optics I are optics of classical design and the radius of curvature of the cathode is R = 125 mm. The images resulting from it are subject to astigmatism and to curvature of the field. In the case of optics II* (sic), we utilized a cathode having a radius of curvature of R = 80 mm, which results in a field curvature which is somewhat smaller. The astigmatism observed with optics I is primarily due to the fact that the various electrodes are off center. In the case of optics II, it was manufactured in such a way that the electrodes were centered as close as possible. In particular, the anode and the neutral electrode are centered with an accuracy on the order of 1/100 mm. As far as the cathode is concerned, only a centering to within 1/10 mm is possible, because it is necessary to maintain sufficient

*Translator's Note: Illegible in foreign text.

play between it and its centering ring in order to take into account the difference in expansion during the first oven drying. Experiments made with these optics, using Foucault test patterns, show that astigmatism exists in addition to the fact that the cathode is off-center. This is possibly due to parasitic magnetization of the * rings P which are provided for centering the electrodes.

*Translator's Note: French word dilver; probably misprint.

CHAPTER I

DISTORTION DERIVED FROM THE SPECTRA CHARACTERISTICS /2

Presentation of the Spectra:

Figure 1 shows an electrographic plate which has a recording of a spectrum of mercury. Two features can be used to study the distortion: the spectrum edges and the interference rings.

The edges of the spectrum have been made as sharp as possible by limiting the slit height by knife edges located essentially in its plane. The small size of the spectral region and the properties of the objective of the photographic chamber did not require the camera to be inclined with respect to the axis of the optical beam. This is why the edges of the spectrum must be two parallel lines. Photographic checks over a much more extensive spectral region showed very well that the distortion of the optics of the spectrograph could be neglected. The study of the curvature of the edges on the electronographic image will make is possible to characterize the distortion using several hypotheses, which are quite restrictive.

The interference rings are due to the insertion of a Pérot-Fabry etalon* in the beam, before it enters the spectrograph. In a certain sense, the interference rings constitute a micrometer which will be used to measure the shifts of the absorption lines of the solar spectrum. The intersections of the interference rings and the absorption lines form figures which are complementary to the ring cross sections at infinity and produced by the spectrograph slit and for emission lines having the same wavelength. The interference figure is

*Translator's Note: This is a dispersive device.

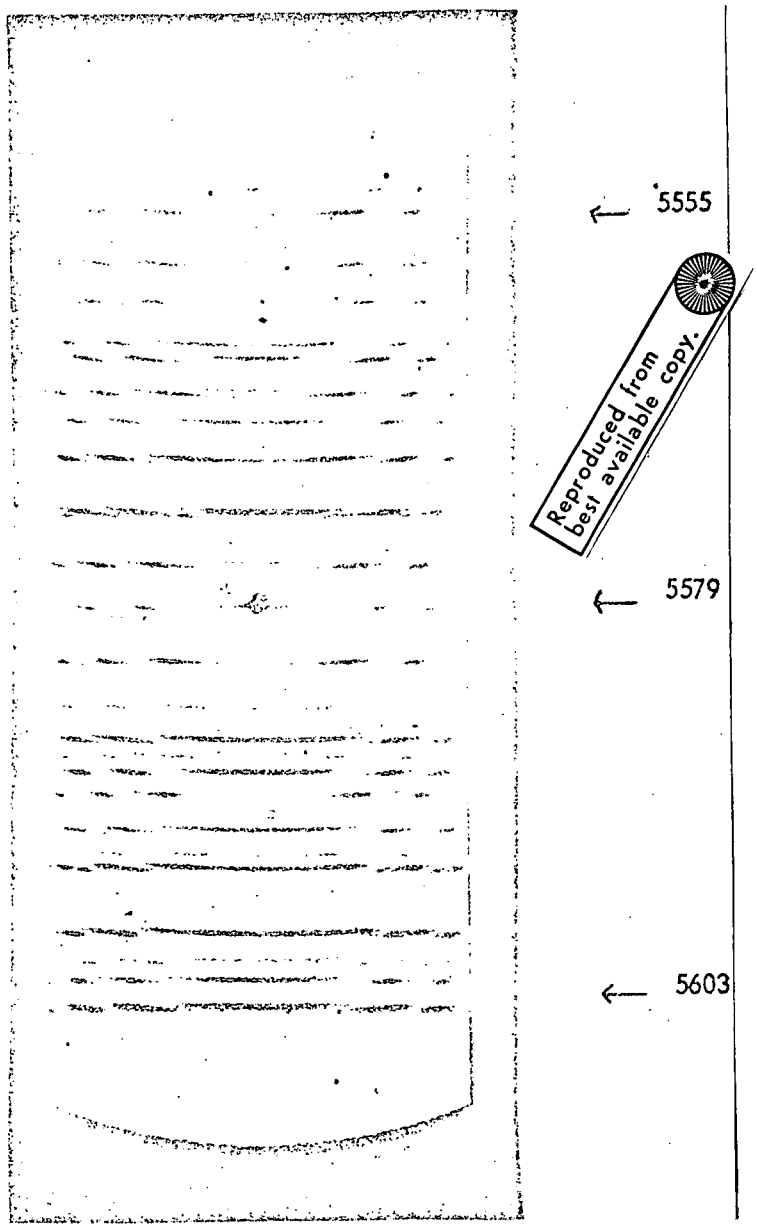

Figure 1. Spectrum of Mercury taken with the electronic camera. 37 minute exposure. October 13, 1963, from 4:45 to 5:22 U.T. (enlarged ten times).

strongly modified when the line is slightly shifted. These problems have been studied in [3]. In the present case, it will be sufficient to know that the spectral line spacing must vary linearly with the abscissa measured along the spectrum on a line parallel to the edges of the spectrum. The measurements of the spacing on the plate will therefore give us information on the distortion .

We will now study these two methods.

Size of the Spectra:

Let a and b be the edges of the image spectrum (Figure 2). L is the variable size of the spectrum, OX and OY are axes. OX is parallel to the dispersion direction, and OY is perpendicular to this direction. We will assume that the distortion has a center C and that any distance D measured from C is related to the distance D of the image without distortion, with the same enlargement as C, by the formula

$$D = d(1+\alpha d^2).$$

Further on we will see whether this hypothesis is an acceptable one, using the analysis of the residuals.

It is clear that the minimum size L corresponds to the abscissa of C, /4
X_c. The ordinate of C, Y_c, can be derived from the curvature of the edges a and b. Finally, α can be calculated from the curvatures of a and b. We will now carry out this procedure in detail.

Let $M(X, Y_i)$ be a point of the edge i (i = a or b); $m(x, y_i)$ — the corresponding point of the undistorted image, and the axes Ox and Oy are identical with OX and OY. The positions of m are:

for the edge a: $y_a = p$,

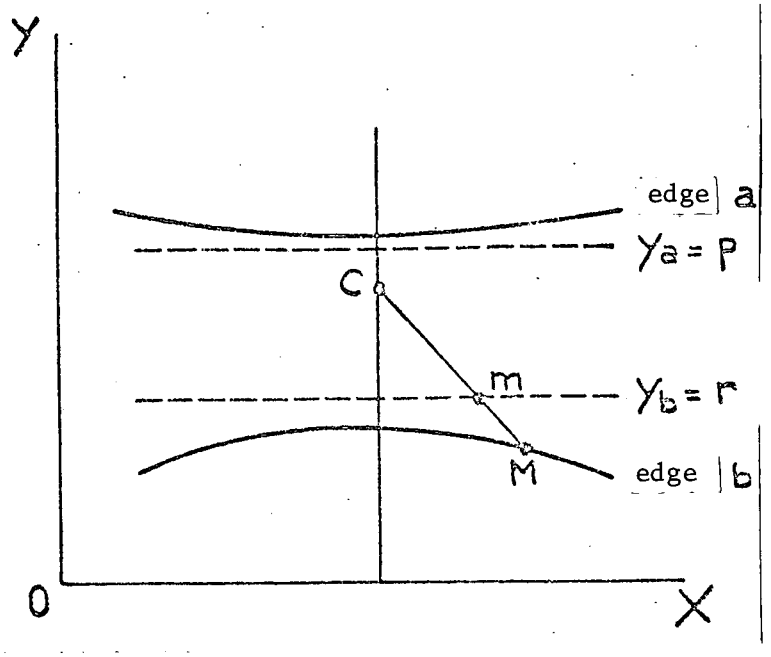

Figure 2

for the edge b: $y_b = r$.

We have $$\frac{Y_a - Y_c}{p - Y_c} = \frac{CM}{Cm} = 1 + \alpha \overline{Cm}^2$$

or

$$Y_a - Y_c = (p - Y_c)\left\{1 + \alpha[(X - X_c)^2 + (Y_a - Y_c)^2]\right\},$$

by making the substitution x for X, y_a for Y_a in the term involving α, i.e., ignoring terms with α^2. It follows from this that,

$$Y_a = p + \alpha (p - Y_c)\ [(X - X_c)^2 + (Y_a - Y_c)^2]. \qquad (1)$$

In the same way, we obtain for the edge b

$$Y_b = r + \alpha (r - Y_c)\ [(X - X_c)^2 + (Y_b - Y_c)^2]. \qquad (2)$$

By setting:

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$$P = p + \alpha (p - Y_c)^3,$$
$$R = r + \alpha (r - Y_c)^3,$$
$$\pi = \alpha (p - Y_c),$$
$$\rho = \alpha (r - Y_c),$$
$$\chi = X - X_c,$$

we will write (1) and (2) in the form:

$$Y_a = P + \pi\chi^2 \quad (3)$$
$$Y_b = R + \rho\chi^2, \quad (4)$$

up to α^2 terms and setting $p = Y_a$ and $r = Y_b$ in the α term.

The measurement results in Y_a and Y_b as a function of X. The first step of the calculation is to find the abscissa X_c corresponding to the minimum of $Y_a - Y_b$. We will then have Y_a and Y_b as a function of X and it will be necessary to solve the two systems of overspecified equations of the types (3) and (4). The unknowns are α, Y_c, p and r. We will proceed using successive approximations applying the condition of least squares to the residuals of the ordinates (which amounts to assuming that the accuracy of the measurements of the ordinates is the same everywhere).

The first step consists of setting $p_1 = P$, $r_1 = R$, i.e., of considering P and R as independent of α. We will obtain the following, if [ ] indicates the sums of the corresponding quantities in each of the Equations (3) or (4) and n is the number of equations

$$P = p_1 = \frac{[Y_a][\chi^4] - [\chi^2 Y_a][\chi^2]}{n[\chi^4] - [\chi^2]^2}, \quad \pi = \frac{n[\chi^2 Y_a] - [\chi^2][Y_a]}{n[\chi^4] - [\chi^2]^2}$$

Similar expressions will be obtained for r_1 and ρ.

Taking the relationship

$$\pi(r - Y_c) = \rho(p - Y_c),$$

into account we find

$$(Y_c)_1 = \frac{\pi r_1 - \rho p_1}{\pi - \rho} \quad , \quad \alpha_1 = \frac{\pi}{p_1 - Y_c} \qquad (5)$$

We will then have improved values of p and r: p_2 and r_2 are given by:

$$p_2 = p_1 - \alpha_1 [p_1 - (Y_c)_1]^3 \quad , \quad r_2 = \cdots\cdots ,$$

Then for Y_c and for α, $(Y_c)_2$ and α_2, are given by expressions similar to (5). One iteration will usually suffice in practice.

According to the graph shown in Figure 3, which shows $Y_a - Y_b$ as a function of X, we will have $X_c = 9.20$ mm. A small inclination of the X axis with respect to the micrometer screw can be detected, because for values of X which are symmetrical with respect to X_c, the Y_a or the Y_b are not equal. In addition, we set $Y'_a = Y_a - 11{,}500$, $Y'_b = Y_b - 17{,}000$ for simplification. We obtain the following if we take the inclination correction into account: /7

X	Y'_a	Y'_b
- 4,2	0,094	0,111
- 3,7	097	103
- 3,2	102	095
- 2,7	104	090
- 2,2	104	083
- 1,7	105	079
- 1,2	110	079
- 0,7	111	076
- 0,2	111	077
+0,3	111	075
+0,8	110	076
+1,3	110	079
+1,8	109	082
+2,3	108	091

n =	18
$[\chi^2]$ =	121,17
$[\chi^4]$ =	1463,1
$[Y'_a]$ =	1,878
$[Y'_b]$ =	1,610
$[\chi^2 Y'_a]$ =	11,992
$[\chi^2 Y'_b]$ =	12,167

Example of a numerical application. Plate of Venus on July 25, 1961, plate No. 7, optics I. All the measurements are expressed in millimeters.

X	Y_a	Y_b	magnitude $Y_a - Y_b$
5,0	11,591	17,108	5,517
5,5	595	101	506
6,0	600	093	493
6,5	602	088	486
7,0	603	082	479
7,5	604	078	474
8,0	609	078	469
8,5	611	076	465
9,0	611	077	466
9,5	611	075	464
10,0	610	076	466
10,5	611	080	469
11,0	610	083	473
11,5	609	092	483
12,0	607	097	490
12,5	601	102	501
13,0	598	109	511
13,5	595	115	520

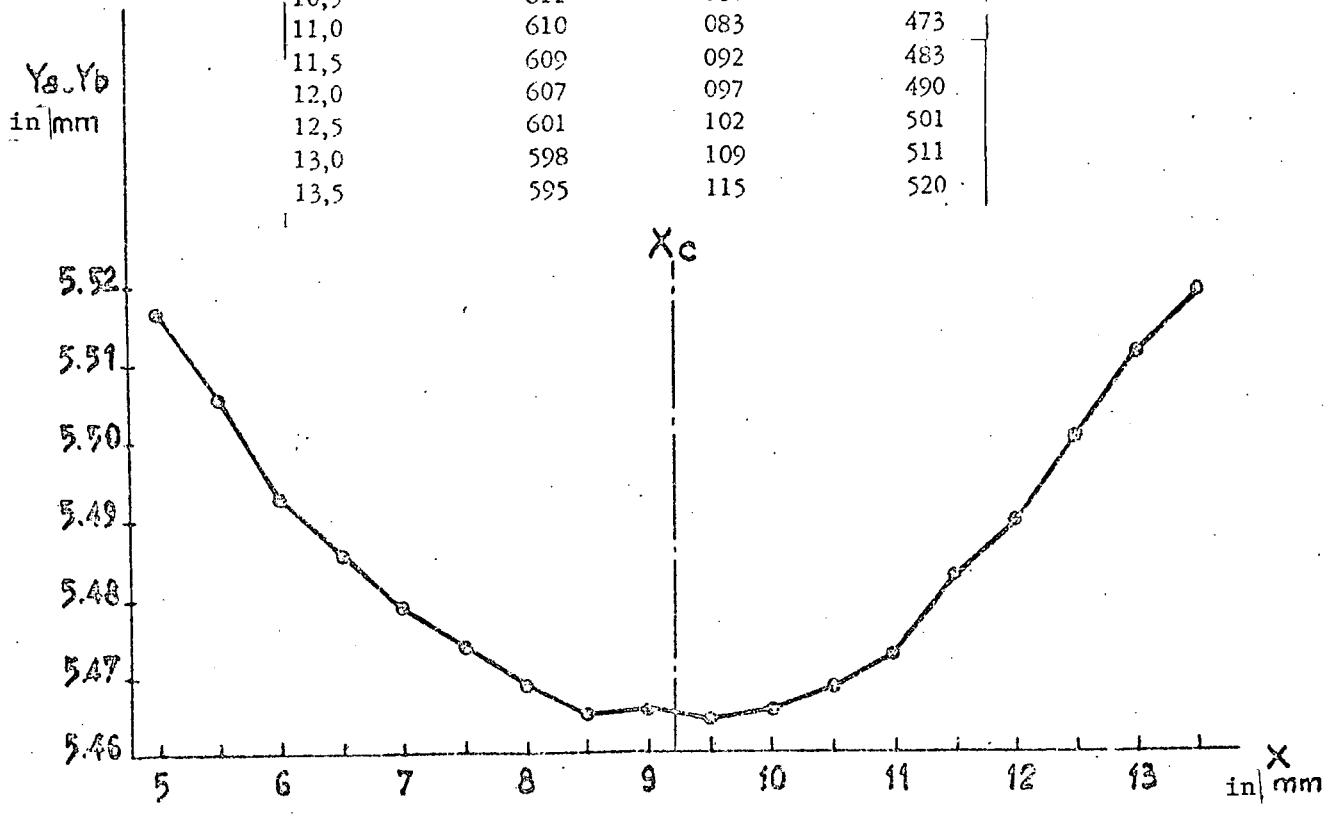

Figure 3. Variation of the quantity $Y_a - Y_b$ of the spectrum as a function of the abscissa X. Y_c is the abscissa of the distortion center.

+2,8	105	095
+3,3	099	100
+3,8	096	107
+4,3	092	112

From this we can derive the values of the unknown quantities π, ρ, p', r' ($p' = p_1 - 11{,}500$, $r' = r_1 - 17{,}000$)

$$\pi = -1{,}009\ 10^{-3}, \qquad p' = 0{,}111, \qquad p_1 = 11{,}611,$$
$$\rho = +2{,}050\ 10^{-3}, \qquad r' = 0{,}076, \qquad r_1 = 17{,}076.$$

Then we have

$$(Y_c)_1 = 13{,}414, \qquad \alpha = 0{,}560\ 10^{-3}.$$

The iteration results in

$$p_2 = 11{,}614, \qquad r_2 = 17{,}048$$

and

$$(Y_c)_2 = 13{,}406, \qquad \alpha = 0{,}563\ 10^{-3}.$$

As a check, we can calculate the coordinates of the points m which correspond to the measurements of the points M and the size of the undistorted spectrum, using the equation:

$$y_a = Y_a - \alpha (Y_a - Y_c) [(X - X_c)^2 + (Y_a - Y_c)^2].$$

X	coordinates of the edges corrected for distortion y_a	y_b	width without distortion $(y_a - y_b)$	deviations with respect to the average magnitude
- 4,2	11,615	17,048	5,433	- 1
- 3,7	14	48	34	0
- 3,2	16	47	31	- 3
- 2,7	15	48	33	- 1
- 2,2	12	46	34	0
- 1,7	11	46	35	+1
- 1,2	15	49	34	0
- 0,7	15	48	33	- 1
- 0,2	14	50	36	+2
+0,3	14	48	34	0
+0,8	14	48	34	0
+1,3	15	48	33	- 1
+1,8	16	48	32	- 2
+2,3	17	53	36	+2
+2,8	16	52	36	+2
+3,3	13	50	37	+3
+3,8	14	50	36	+2
+4,3	14	47	33	- 1

This table shows that the adjustment is satisfactory. The deviations are distributed irregularly and their mean square is on the order of 1.5 μ. The measurements were limited to 5.1 mm from the center of the plate. Outside this area, because of the field curvature, measurements having an accuracy of 1 micron no longer are justified. This means that the representation by means of a third order formula which involves only one parameter α is therefore satisfactory in the range studied, if an accuracy of 1 micron is required. The same was true for all the spectra measured by this method.

Results of the Width Measurements

The measurements of the width extended over 20 spectra taken during three nights. They were only done for the absorption spectra and not for

the emission reference spectra (iron arc), because the edges are not visible. Tables I and II show the results.

TABLE I. RESULT OF WIDTH MEASUREMENTS. DISTORTION REPRESENTED BY $D = d(1 + \alpha d^2)$, d and D in mm. THE COORDINATES (IN mm) OF THE DISTORTION CENTER WITH RESPECT TO THE PHOTO CATHODE (X_0, Y_0) AND THE WIDTH OF THE UNDISTORTED SPECTRUM, l, ARE GIVEN. OPTICS I.

Date	Plate	α (in 10^{-3})	$X_c - X_0$ in mm.	$Y_c - Y_0$ in mm.	l in mm.
8 July 1961	2	0,598	- 0,82	- 0,25	5,508
	4	577	- 0,95	- 0,49	5,498
	5	536	- 0,90	- 0,90	5,490
	9	612	- 1,00	- 0,70	5,505
	10	697	- 1,05	- 0,42	5,509
	11	556	- 0,98	+ 0,25	5,497
20 July 1961	2	0,581	+ 0,11	- 1,25	5,455
	3	525	+ 0,22	- 1,03	5,464
	5	568	+ 0,17	- 1,21	5,461
	6	547	+ 0,13	- 1,38	5,465
	7	532	+ 0,04	- 1,34	5,469
	8	569	+ 0,14	- 1,24	5,454
	11	573	+ 0,28	- 1,21	5,447
25 July 1961	2	0,552	0,00	- 0,32	5,429
	3	553	- 0,01	- 0,52	5,440
	5	562	- 0,05	- 0,71	5,434
	6	587	- 0,06	- 0,61	5,433
	7	563	- 0,08	- 0,83	5,434
	10	520	0,00	- 0,55	5,438
	11	557	+ 0,03	- 0,62	5,441

TABLE II. AVERAGES PER NIGHT, WITH TYPICAL DEVIATIONS:

8 July 1961	$10^3 \alpha$	=	0,596 ± 0,023	
	$X_c - X_o$	= -	0,95 ± 0,03	(in mm)
	$Y_c - Y_o$	= -	0,42 ± 0,22	(in mm)
20 July 1961	$10^3 \alpha$	=	0,556 ± 0,008	
	$X_c - X_o$	= +	0,16 ± 0,03	(in mm)
	$Y_c - Y_o$	= -	1,24 ± 0,04	(in mm)
25 July 1961	$10^3 \alpha$	=	0,556 ± 0,007	
	$X_c - X_o$	= -	0,02 ± 0,02	(in mm)
	$Y_c - Y_o$	= -	0,59 ± 0,06	(in mm)

Discussion of Width Measurements

Even though this method gives good internal accuracy in the case of sharp plates, such as the ones for July 20 and 25, 1961, it did not seem entirely satisfactory to us.

First of all, the measurements introduce the distortion in variable directions. In the case in which this aberration does not have rotational symmetry, the method leads to results having no clear physical significance. In any case, it is possible to see whether the distortion varies for a series of plates obtained with the same optics.

The second disadvantage consists of the difficulty of measuring the position of the dark edge of the spectrum against the light background of the plate. As soon as the edge ceases to be sharp, (because of the field curvature in the vicinity of the edges of the plate), the judgment of the observer is introduced and brings about systematic errors. The plates taken on July 8 were quite fuzzy along the edge; there is a considerable variation in the values between plates. No other method was able to confirm these variations.

Finally, this method cannot be applied to emission spectra which lack a continuous background. /10

For all these reasons, we attempted to study the distortion by measuring the spacing of the interference rings.

Use of the Interference Lines

The dispersing unit in the spectrograph is a grating. Within the small spectral range in question, the spectrum can be considered as strictly perpendicular. Let λ be the wavelength, κ the dispersion in Å per mm. We will have the following relation for the undistorted spectrum:

$$x - x_0 = \frac{1}{\kappa} (\lambda - \lambda_0), \tag{6}$$

where x_0 is the abscissa of the line having the wavelength λ_0.

The Perot-Fabry etalon only transmits light of wavelengths which satisfy the condition:

$$p\lambda = 2e \cos i, \tag{7}$$

where p is a whole number, e the optical thickness of the etalon, and i is the incidence angle of the beam on the grating grooves. Our device satisfies the equation

$$y - y_0 = \frac{1}{\zeta}, \tag{8}$$

ζ is a constant which depends on the focal length of the objective which projects the rings at infinity on the spectrograph slit and y_0 is the ordinate for i = 0. In addition, we can replace cos i by an expansion limited to the second order. Equation (6) becomes the following, where (7) and (8) have been taken into account

$$p\kappa(x - x_0) + \zeta^2 e(y - y_0)^2 = 2e - p\lambda_0. \tag{9}$$

This is the equation of the interference rings without distortion. They are parabolas with the axis $y = y_0$. Their concave side is directed towards short wavelengths.

We propose to utilize the distances Δx between the interference rings (spacings) measured along lines parallel to the x axis. According to (9) where y is considered to be a constant, we have:

$$p\kappa\, dx = -[\lambda_0 + \kappa(x - x_0)]\, dp.$$

The distance between two consecutive interference rings corresponds to $|dp| = 1$. By replacing p by its value as a function of x and y and by ignoring the sign, we obtain:

$$\Delta x = \frac{[\lambda_c + \kappa(x - x_0)]^2}{2e\kappa\,[1 - 1/2\,\zeta^2(y - y_0)^2]}$$

In the denominator, it is possible to ignore $1/2\,\zeta^2(y - y_0)^2 = \frac{i^2}{2}$

In effect, the maximum value of i is approximately 1/50 rad, and the neglected term is less than 1/5000. This means that a small inclination of the spectrum with respect to the grooves of the measurement device will be completely unnoticeable.

We can set $x_0 = x_c$ and $\lambda_0 = \lambda_c$ (λ_c is the wavelength which corresponds to the abscissa of the distortion center), so that: /11

$$\Delta x = \Delta x_0 + \frac{\lambda_c}{e}(x - x_c) + \frac{\kappa}{2e}(x - x_c)^2, \qquad (10)$$

where $\Delta x_0 = \frac{\lambda_c^2}{2e}$ is the interference ring spacing at the distortion center.

First, the approximate coordinates X_c, Y_c of the distortion center C will be measured (by width measurements, for example). Let M(X) be the intersection of an interference ring with the line parallel to OX passing through

C, and let m(x) be the corresponding point of the image without distortion:

$$X - X_c = x - x_c + \alpha(x - x_c)^3$$

it follows that

$$\Delta X = \Delta x + 3\alpha(x - x_c)^2 \Delta x,$$

This can be written by replacing $x - x_c$ by $X - X_c$, as well as Δx by $\Delta x_0 = \Delta X_0$ in the coefficient of α:

$$\Delta X = \Delta x + 3\alpha(X - X_c)^2 \Delta X_o.$$

Because of (10) we obtain the following, again replacing $x - x_c$ by $X - X_c$:

$$\Delta X = \Delta X_o + \frac{\lambda_c}{e}(X - X_c) + (3\alpha\Delta X_o + \frac{\kappa}{2e})(X - X_c)^2. \qquad (11)$$

In the factor $(X - X_c)^2$, we have approximately:

$$\alpha = 0{,}5\ 10^{-3},\quad \Delta X_o = 0{,}08,\quad \kappa = 0{,}47\ 10^{-6}(4{,}7\ \text{Å/mm}),$$

$e = 4$, where everything is expressed in millimeters. From this we find:

$$3\alpha\Delta X_o = 0{,}12\ 10^{-3}$$
$$\kappa/2e = 0{,}06\ 10^{-6}.$$

The term $\kappa/2e$ is completely negligible. This amounts to assuming that the interference ring spacing varies linearly as a function of x over the range of the plate which is not affected by distortion.

We will therefore attempt to find the solution of the equations of condition:

$$\Delta X = \Delta X_o + \frac{\lambda_c}{e}(X - X_c) + m(X - X_c)^2. \qquad (12)$$

where the unknowns are ΔX_0, λ_c/e and m. Then α will be calculated from $\alpha = m/3\Delta X_0$. Even though λ_c/e is known a priori to a good approximation, it is better to let it be one of the unknowns, because this term depends strongly on the error in estimating X_c.

We will not calculate the error in α here. This error is the result of estimation errors in X_c and Y_c. These errors are only small compared with the accuracy with which α is determined.

The interference rings cannot be seen on the emission spectra, but it is possible to slightly modify the method and measure the line positions taking /12
the fractional excess (approximately 0.01) of the interference order into account. We will not discuss the details here. Neither will we describe the measurement process and the precautions taken to reduce accidental and systematic errors, especially those connected with periodic errors of the micrometer screw.

Numerical Example. July 20, 1961, Plate No. 1, Iron Spectrum, Optics I.

$X - X_c$ in mm	ΔX in μ	ΔX calculated in μ	residuals in μ
−4,76	42,20	42,21	−0,01
−3,87	41,56	41,53	+0,03
−2,93	40,96	40,94	+0,02
−1,87	40,27	40,41	−0,14
−0,71	40,07	40,00	+0,07
+0,30	39,88	39,80	+0,08
+1,17	39,69	39,73	−0,04
+2,15	39,85	39,78	+0,07
+3,30	39,86	40,00	−0,14
+4,28	40,40	40,33	+0,07

The solution gives:

$$m = 0{,}0674 \quad 10^{-3}, \quad \Delta x_o = 39{,}846, \text{ and } \alpha = \frac{m}{3\Delta X_o} = 0{,}563 \quad 10^{-3}.$$

The third column gives ΔX calculated from values found for the unknowns. The fourth column gives the residues ΔX observed — ΔX calculated.

The calculation also gives the standard deviation of an isolated value of ΔX : $\sigma_{\Delta X} = 0.093$ and the standard deviation of α,

$$\sigma_\alpha = 0{,}033 \ 10^{-3}.$$

Results

Using this method, we studied sixteen plates of Venus and of iron, taken July 20 and 25, 1961 and using optics I. We also studied 25 plates of mercury and of iron, taken over three nights in October, 1963, and using optics II.

a - Optics I - The following give the results using these optics. The number in parentheses indicates the plate numbers which have the iron spectrum.

Plates of July 20

Plate	α in 10^{-3}
(1)	0,568
(4)	0,471
(9)	0,520
(10)	0,490
(12)	0,545

Plates of July 25

Plate	α in 10^{-3}
(1)	0,544
2	0,561
3	0,525
(4)	0,500
5	0,614
6	0,553
7	0,497
(8)	0,485
10	0,516
11	0,531
(12)	0,543

In this way, we obtain the average (using the standard deviations): /13

for July 20, 1961 $\alpha = 0{,}518 \ 10^{-3} \pm 0{,}017 \ 10^{-3}$

for July 25, 1961 $\alpha = 0{,}534 \ 10^{-3} \pm 0{,}013 \ 10^{-3}$

for the combination $\alpha = 0{,}529 \ 10^{-3} \pm 0{,}009 \ 10^{-3}$

b - Optics II - The results obtained are the following:

1st series		2nd series		3rd series	
5, 6, 8 Oct. 1963		11, 12, 13 Oct. 1963		14, 15 Oct. 1963	
Plate	α in 10^{-3}	Plate	α in 10^{-3}	Plate	α in 10^{-3}
(1)	0,588	(2)	0,513	(1)	0,508
(3)	0,559	3	0,520	(3)	0,541
(4)	0,572	(4)	0,467	(4)	0,517
(6)	0,534	(5)	0,483	5	0,580
7	0,532	6	0,564	(6)	0,492
(8)	0,522	(7)	0,491	7 }	
(9)	0,439	8	grid image	8 }	grid
10	0,542	(9)	0,480	9 }	images,
11	0,565	10	0,581	10 }	see
(12)	0,529	(11)	0,492	11 }	following
		12	0,572	12 }	chapter

from which the following averages are obtained:

first series: $\alpha = 0{,}538\ 10^{-3} \pm 0{,}013\ 10^{-3}$

second series: $\alpha = 0{,}516\ 10^{-3} \pm 0{,}013\ 10^{-3}$

third series: $\alpha = 0{,}528\ 10^{-3} \pm 0{,}016\ 10^{-3}$

The general average for optics II is:

$$\alpha = 0{,}527\ 10^{-3} \pm 0{,}008\ 10^{-3}.$$

Generalization of the Method. When common spectra are being evaluated without overlapping interference rings, the method can be used provided that a sufficient member of wavelengths and the dispersion law of the spectrograph are known. This procedure results in a rapid grid check of the distortion coefficient without wasting any plates.

CHAPTER II

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STUDY OF THE DISTORTION AND THE ENLARGEMENT BY GRID PLATES

The image of a grid is projected on the photo cathode of the electronic camera. It is transformed into an electronic image which is collected on an electronographic plate. In the following, we will call it simply the image. The grid and its image cannot be directly compared, because the optics of the projector has its own aberrations and its enlargement factor is poorly defined. In a preliminary experiment, a photographic plate is placed at the location of the photo cathode and plates of the grid are obtained which we will simply call the object. We took all the precautions to ensure that the projector is in the same position during all the experiments. Its optical setting is not changed and the plane of the optical image is practically parallel to the plane of the photo cathode. It is mounted at three points on the sealing plate of the camera tube. We should also note that the projection object used was open to f/11 (in order to limit the aberrations introduced by the meniscus on which the photo-sensitive layer is deposited) and the position of the image of the grid cannot be strictly defined because of the depth of field. This procedure will therefore not result in an accurate value of the enlargement, but nevertheless it will allow the study of very small variations.

For a given set of optics, the distortion coefficient is not perfectly defined, because various effects can intervene and change its value. In particular, we studied the influence of the following:

1/ the successive assembly and disassembly of the electronic optics, during which its various components are not placed in their respective

positions with the required accuracy.

2/ a small variation of the voltages applied to the electrodes. The electrometers used for measuring these voltages are in the 1% class. The study which we carried out showed that their fidelity did not make it possible to readjust the focalization relationship to a high degree of accuracy.

3/ the position of the electronographic plate. In spite of the care during the manufacture of the cylinder, successive photographic plates did not have exactly the same position with respect to the electronic image.

4/ the properties of the emulsion support: glass or plastic.

Theoretical Solution

Just as in previous cases, we did not carry out the measurements up to the edge of the photocathode. In this region, the image of the line is affected by an adjustment error due to the field curvature. In addition, imperfect centering of the electrodes produces a decentering astigmatism which introduces an uncertainty in pointing. Under these conditions, we found it unnecessary to push the development of the distortion above the third order.

We assumed that a point C exists where the distortion is zero, but we considered the possibility of a variation of the distortion and the enlargement with direction away from C. If D is the distance on the image reckoned from C, and if d is the corresponding distance on the object, we assume that

$$D = kd(1 + \beta d^2).$$

k is an unknown factor which defines the enlargement at C, β is a coefficient which cannot be directly compared with the coefficient α calculated in the first chapter, where we were limited to adjustment of the image without

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determining the enlargement. Assuming the origin to be at the distortion center, we had in the first chapter:

$$X = x(1 + \alpha x^2).$$

In the present case we have:

$$Z = kz(1 + \beta z^2).$$

If we set X equal to Z and taking z = kz into account, we find:

$$\alpha = \frac{\beta}{k^2}.$$

In order to carry out comparisons, we will give values of α but not values of β in the numerical results. In theory, α will not be independent of k, but in practice the relative error of k is negligible compared with the errors in α or β.

The general solution consists of measuring the coordinates of all the intersections of the object and of the image. Then all the unknowns of their comparison are determined. These unknowns are eight in number, even assuming that the totality of deformations of the electronic image is reduced to 1) affinity, 2) a distortion reduced to the third order assuming two rectangular axes of symmetry and an elliptical distribution as a function of direction. These unknowns consist of: the two coordinates of C, the affinity direction and the direction of the greatest distortion, the two extreme values of α, the distortion coefficient and the two extreme enlargements.

In order to facilitate the calculations and their programming for a computer, we used a simplified method based on the spectral line measurements.

Let us assume that it is required to determine a line passing through C on the image. Let Z be the abscissas measured along this line from an arbitrary

origin. Let Z_c be the abscissa of C (unknown), z and z_c the corresponding abscissas on the object. If k is the enlargement of C, we have:

$$Z = Z_c + k[(z - z_c) + \beta(z - z_c)^3] \quad (1)$$

We will compare the intervals $Z_2 - Z_1$ to $z_2 - z_1$.

$$Z_2 - Z_1 = k(z_2 - z_1)[1 + \beta\{(z_2 - z_c)^2 + (z_1 - z_c)(z_2 - z_c) + (z_1 - z_c)^2\}] \quad (2)$$

Then (1) can be written as:

$$z - z_c = \frac{Z - Z_c}{k} \quad \frac{1}{1 + \beta(z - z_c)^2} = \frac{Z - Z_c}{k}[1 - \beta(z - z_c)^2]$$

In the coefficient of β, we will simply write $z - z_c = \frac{Z - Z_c}{k}$, which amounts to ignoring the terms in β^2. It follows that:

$$\chi = \frac{Z_2 - Z_1}{z_2 - z_1} = k[1 + \frac{\beta}{k^2}\{(Z_2 - Z_c)^2 + (Z_2 - Z_c)(Z_1 - Z_c) + (Z_1 - Z_c)^2\}]. \quad (3)$$

For simplification, let us write χ as a function of the unknowns: $\chi = f(k, \beta, Z_c)$. χ is obtained from the measurements. Several values of χ can be obtained, and the most probable value of the unknowns can be found. In order to apply the method of least squares in an easy way, it is necessary for the equations of condition to be linear. Therefore we will assume that approximate values k_0, β_0, $(Z_c)_0$ of k, β and Z_c are known. In practice, we will assume that k_0 is the enlargement derived from a comparison of the image and the object, ignoring the distortion. We will assume that Z_c is the coordinate of the image center of the photo cathode. If we completely ignore the order of magnitude of β_0, we can adapt the value obtained from a first calculation where we set $\beta_0 = 0$. We will calculate $\chi_o = f(k_o, \beta_o, (Z_c)_o)$, and then set:

$$\xi = \chi - \chi_o, \quad x = k - k_o, \quad y = \beta - \beta_o, \quad z = Z_c - (Z_c)_o.$$

from which it follows that:

$$\xi = ax + by + cz,$$

$$a = \frac{\partial \chi}{\partial k} = 1 - \frac{\beta_o}{k_o^2} \{(Z_2 - (Z_c)_o)^2 + (Z_2 - (Z_c)_o)(Z_1 - (Z_c)_o) + (Z_1 - (Z_c)_o)^2\},$$

$$b = \frac{\partial \chi}{\partial \beta} = \frac{1}{k_o} \{ \ldots\ldots\ldots \},$$

$$c = \frac{\partial \chi}{\partial Z_c} = 3 \frac{\beta_o}{k_o} [2(Z_c)_o - (Z_2 + Z_1)].$$

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Measurement and Data Reduction Procedure

We used a grid having 0.5 mm spacing, which G. Wlérick was kind enough to lend to us. Let us consider the directions x and y which are the two orthogonal directions of the grid. Let Dx and Dy be the ensembles of the lines corresponding to the lines x = constant and y = constant.

First the spacing of the Dy along an axis Dx passing through the intersection of the two lines closest to the image of the center of the photocathode will be measured. Using the described method of comparison with the object, the values of k, β and X_c are derived. They are only approximate ones, because the intersection used as the center of the plate does not in general coincide with the distortion center. The value of X_c is quite accurate for the selection of the line Dy, along which it is required to measure the distances between the Dx. We then obtain definite values of the unknowns in the direction Y: k_y, β_y and Y_c. If necessary, the measurement along the first direction must be carried out again so that Dx passes as close as possible to the point (X_c, Y_c) and we obtain k_x, β_x, X_c.

We also measured the diagonals of the squares in order to obtain values of the unknowns in four directions: k_z β_z and Z_c; k_t, β_t and T_c.

The calculations were all made on the IBM 650 computer of the Paris Observatory.

Results

The spectra of Mercury should have sufficient density to make the measurements easy. Therefore, in the case of the plates for Mercury as well as the plates for the grids, we were led to use Definix radiographic plates, whose sensitivity is three to four times higher than that of ionographic plates Ilford G5.

Twenty-one grid images, all obtained using optics II, were measured in four directions. In three series of experiments, we changed the ratio of the voltages applied to the camera electrodes. We also performed an experiment series with constant voltages in order to verify whether the distortion and the enlargement were stable under these conditions. The standard deviations were calculated. We will briefly state that they are overestimated because the error in the positioning of a line simultaneously affects the local enlargement for two adjacent squares.

Table III gives the results.

Study of the Results /17

a - Enlargement

The voltages and the direction of measurement often vary for the same series of plates.

We will call the average of the enlargements along the four directions the average enlargement $\bar{G}$. R_0 is the ratio of the voltages which provides the best focusing on the plate. R is the ratio of the variable voltages. Figure 4 shows the variation of $\bar{G}$ as a function of $\Delta R = R - R_0$. Linear adjustment by calculation gives the result shown in Figure 4, as well as the standard deviations.

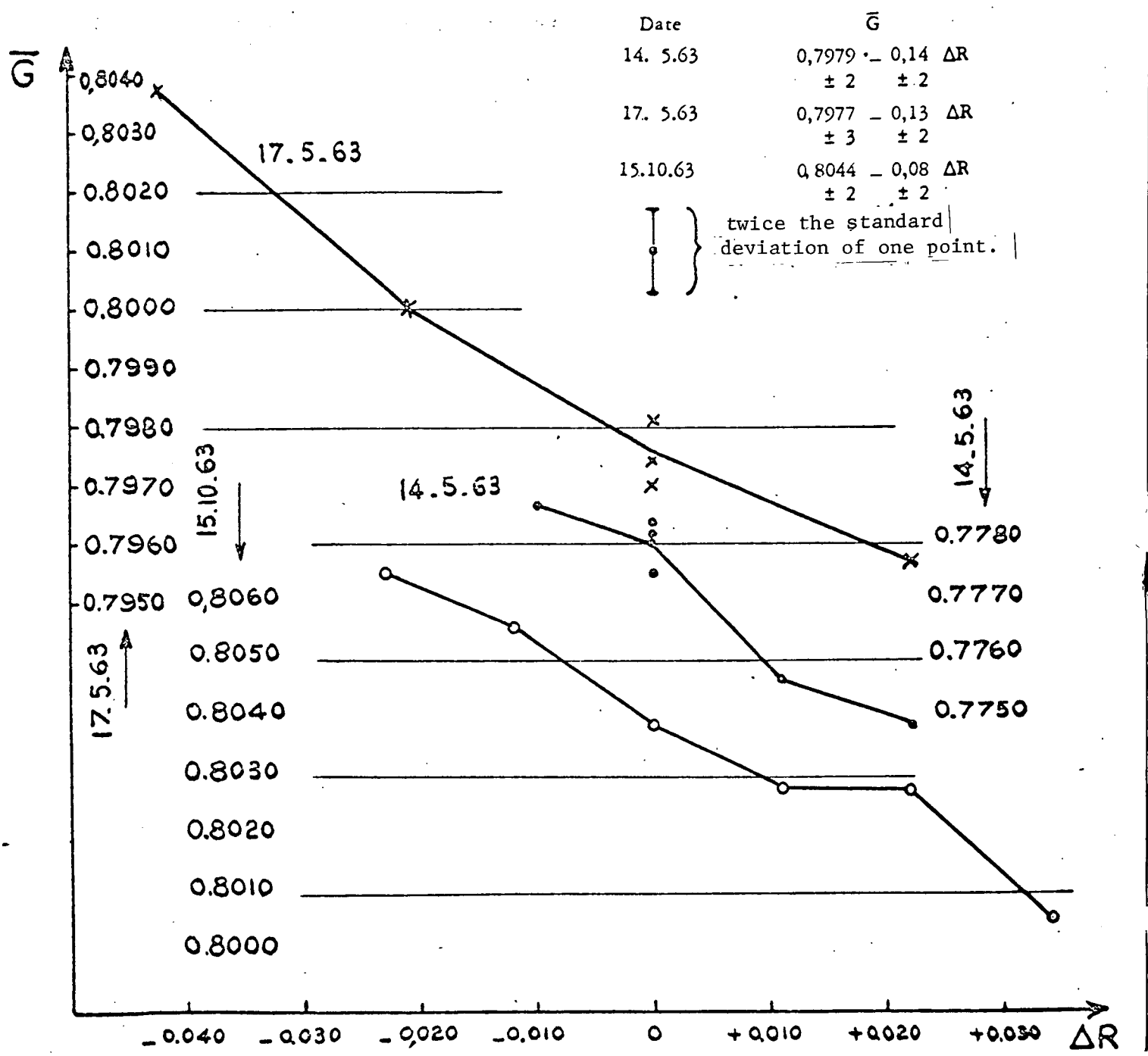

Figure 4. Variation of the average enlargement $\bar{G}$ as a function of the voltage ratio: $\Delta R = R - R_0$. R variable ratio, R_0 ratio which results in the best image. We have indicated a standard deviation of one value of $\bar{G}$.

In order to study the enlargement as a function of direction, we took the average G_i of the enlargement for all the plates out of the same series in the direction i. The following table shows the values of G_i. /18

Date	G_x	G_y	G_z	G_t
14. 5.63	0,7999	0,7948	0,7977	0,7968
17. 5.63	0,8006	0,7970	0,7991	0,7979
15.10.63	0,8010	0,8065	0,8037	0,8036

The standard deviation of a value of G_i is on the order of 0.006 (it is derived from the standard deviation of $\bar{G}$). Figure 5 shows the affinity introduced by the electronic camera. It can easily be seen from Table III that, during the course of one experiment, this affinity is not modified by voltage variations.

TABLE III. RESULT OF GRID MEASUREMENTS, IN THE DIRECTIONS x, y, z, and t. k: ENLARGEMENT OF THE CENTER OF DISTORTION, WITH ITS STANDARD DEVIATION σ_k; α: COEFFICIENT OF DISTORTION, WITH ITS STANDARD DEVIATION σ_α; c: COORDINATES OF THE DISTORTION CENTER, WITH THEIR STANDARD DEVIATIONS σ_c, WITH RESPECT TO THE CENTER OF THE IMAGE OF THE PHOTO CATHODE; R: RATIO OF THE ELECTRODE VOLTAGES WHEN IT IS VARIED.

Date and No. of plate	Dir.	k	σ_k	$10^3\alpha$	$10^3\sigma_\alpha$	c (in mm)	σ_c (in mm)	R	Remarks
13. 2.63	x	0,8227	0,0018	0,461	0,058	+0,06	0,18		Experiment at the Haute-Provence Observatory.
	y	8202	13	494	83	−0,18	19		
	z	8206	21	536	64	−0,02	18		
	t	8218	22	482	64	+0,26	21		
14. 5.63	x	0,7981	0,0013	0,468	0,058	+0,28	0,16		Experiment carried out at Paris.
	y	7913	11	625	75	+0,45	28	2,925	
4	z	7951	16	568	57	+0,52	28		
	t	7946	15	510	47	−0,35	22		
	x	7983	9	453	42	+0,37	12		
	y	7941	9	560	49	+0,28	26		
5	z	7952	16	582	50	+0,64	28	2,914	
	t	7953	11	584	40	−0,04	18		
	x	8007	16	408	69	+0,24	18		
	y	7948	10	568	57	+0,80	26		
6	z	8002	11	578	37	+0,78	18	2,903	
	t	7972	11	575	42	−0,07	23		

TABLE III (Continued)

Date and No. of plate	Dir.	k	σ_k	$10^3\alpha$	$10^3\sigma_\alpha$	c (in mm)	σ_c (in mm)	R	Remarks
7	x	8033	18	362	79	+0,23	22	2,893	
	y	7960	08	584	46	+0,63	24		
	z	7982	20	621	71	+0,38	41		
	t	7972	16	710	78	+0,40	41		
10	x	7992	13	495	56	+0,23	14	2,903	
	y	7952	11	547	74	+0,63	33		
	z	7980	16	558	108	+0,39	38		
	t	7974	13	579	88	+0,07	31		
11	x	7999	10	471	44	+0,24	11	2,903	
	y	7961	09	575	50	+0,57	22		
	z	7987	09	520	50	+0,68	13		
	t	7978	21	525	136	−0,01	37		
12	x	7999	13	511	58	+0,24	16	2,903	
	y	7963	10	547	55	+0,67	29		
	z	7988	20	488	57	+0,86	47		
	t	7982	25	495	189	−0,22	56		
17. 5.63 5	x	0,7975	0,0012	0,502	0,053	+0,13	0,15	2,925	Experiment carried out in Paris
	y	7930	06	562	35	+0,36	12		
	z	7963	07	535	35	+0,52	10		
	t	7956	10	518	43	−0,09	17		
6	x	8024	10	490	45	+0,25	15	2,882	
	y	7987	09	562	49	+0,50	19		
	z	8000	17	569	78	+0,38	27		
	t	7988	14	562	55	−0,20	18		
7	x	8055	09	510	40	+0,07	12	2,860	
	y	8023	09	515	51	+0,57	25		
	z	8047	12	497	59	+0,67	22		
	t	8030	18	490	73	−0,39	35		
10	x	7994	12	480	55	+0,12	18	2,903	
	y	7959	06	576	35	+0,34	15		
	z	7977	15	532	67	+0,40	23		
	t	7964	16	583	72	−0,03	25		
11	x	7986	09	518	41	+0,22	13	2,903	
	y	7954	10	580	53	+0,25	25		
	z	7974	16	532	71	+0,33	24		
	t	7966	18	540	78	+0,10	27		
12	x	8004	10	466	48	+0,09	15	2,903	
	y	7967	12	532	69	+0,54	29		
	z	7983	14	488	61	+0,62	21		
	t	7970	18	555	78	−0,07	27		

Date and No. of plate	Dir.	k	σ_k	$10^3\alpha$	$10^3\sigma_\alpha$	c (in mm)	σ_c (in mm)	R	Remarks
13.10.63	x	8041	11	506	51	−0,02	13		Experiment carried out at the Haute Provence Observatory
	y	8085	18	512	90	+0,21	22		
	z	8072	21	502	71	+0,10	23		
	t	8061	18	540	55	−0,18	19		
15.10.63	x	7990	21	443	104	−0,15	15		
7	y	8030	12	645	62	+0,07	17	3,091	Experiment carried out at the Haute Provence Observatory
	z	8001	25	570	95	+0,02	27		
	t	8007	17	565	58	−0,27	17		
8	x	7991	14	535	69	−0,19	12	3,079	
	y	8060	11	542	49	+0,20	14		
	z	8035	27	507	74	−0,03	26		
	t	8024	15	488	48	−0,24	15		
9	x	7997	13	537	64	−0,27	16	3,068	
	y	8061	17	490	82	+0,05	24		
	z	8035	23	545	70	+0,22	22		
	t	8018	16	578	62	−0,16	13		
10	x	8008	18	577	84	−0,34	11	3,057	
	y	8066	14	556	71	+0,14	19		
	z	8040	16	552	46	+0,23	15		
	t	8040	26	509	79	−0,17	25		
11	x	8033	16	497	76	−0,27	11	3,045	
	y	8088	16	532	75	+0,14	23		
	z	8049	18	587	54	−0,05	17		
	t	8057	29	500	88	−0,09	28		
12	x	8043	15	527	71	−0,35	11	3,034	
	y	8083	13	638	64	+0,14	21		
	z	8063	32	547	97	−0,07	35		
	t	8071	20	505	68	−0,29	21		
					19				

Plates 6, 10, 11, 12 of May 14, 1963 and 10, 11, 12 of May 17, 1963 /21
were taken with the same voltage ratio. In each one of these series, no significant variation in the enlargement from one plate to another can be observed. This indicates that the voltage stability is sufficient and that mechanical defects of the cylinder holding the plates have no effect.

It can also be seen that the measurement series of May 14 and May 17 lead to the same results. However, the change is evident if we consider the

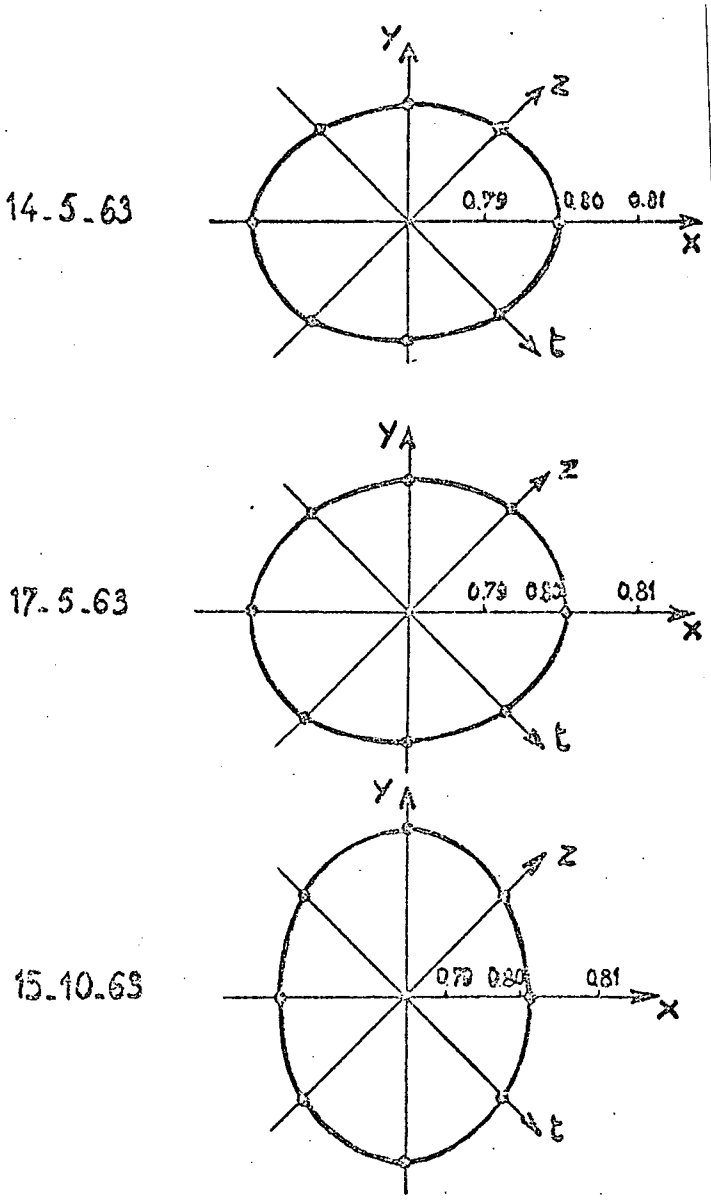

Figure 5. Variation of the enlargement as a function of direction. Ox points in the height direction of the shutter aperture.

series of October 15.

The following are causes for anisotropy in the enlargement: lack of revolution symmetry of the focusing field, inclination of the test pattern projector and anisotropic deformation of the electronographic emulsion base. The precautions taken during the installation of the projector mean that it is possible to eliminate the inclination hypothesis, at least up to the order of magnitude required to explain the results.

The deformations of the emulsion base are to be feared more if plastic films are used. The image is recorded when the film is at a very low temperature. It is measured under standard temperature and humidity conditions. We carried out an experiment, which we shall not describe in detail here, in which we successively recorded the grid image on plates (glass) on two films, cut from the same base in such a way that their long edges make a right angle with respect to

each other. It was clear that the films are deformed and result in an enlargement anisotropy of the same order as the one which appeared in the preceding results. Nevertheless, some points of the experiment remain unclear, because the two films cut at right angles are not deformed in the expected way, if reference is made to a glass plate. Does glass also have an anisotropic deformation? We will carry out further experiments on this subject.

The deformations of the emulsion bases have no perceptible influence on the determinations of the distortion coefficient which we will now study.

b - Distortion

Just as in the case of the enlargement, we attempted to find a variation of the average coefficient $\bar{\alpha}$ taken for the four directions of a plate as a function of the voltage ratio. We obtained Figure 6 and the following expressions (with the standard deviations).

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Date	$\bar{\alpha} \times 10^6$
14. 5.63	$539 - 2\,(100\,\Delta R)$ $\pm 11 \pm 8$
17. 5.63	$531 + 3\,(100\,\Delta R)$ $\pm 8 \pm 4$
15.10.63	$542 - 8\,(100\,\Delta R)$ $\pm 7 \pm 4$

There is no correlation of $\bar{\alpha}$ with ΔR.

We will then study the distortion as a function of the direction by taking the average $\bar{\alpha}_i$ over the direction i of all the values of α which belong to the same plate series.

Date	$\bar{\alpha}_x \times 10^3$	$\bar{\alpha}_y \times 10^3$	$\bar{\alpha}_z \times 10^3$	$\bar{\alpha}_t \times 10^3$
14. 5.63	0,453	0,572	0,559	0,568
17. 5.63	0,494	0,555	0,526	0,541
15.10.63	0,519	0,567	0,551	0,524

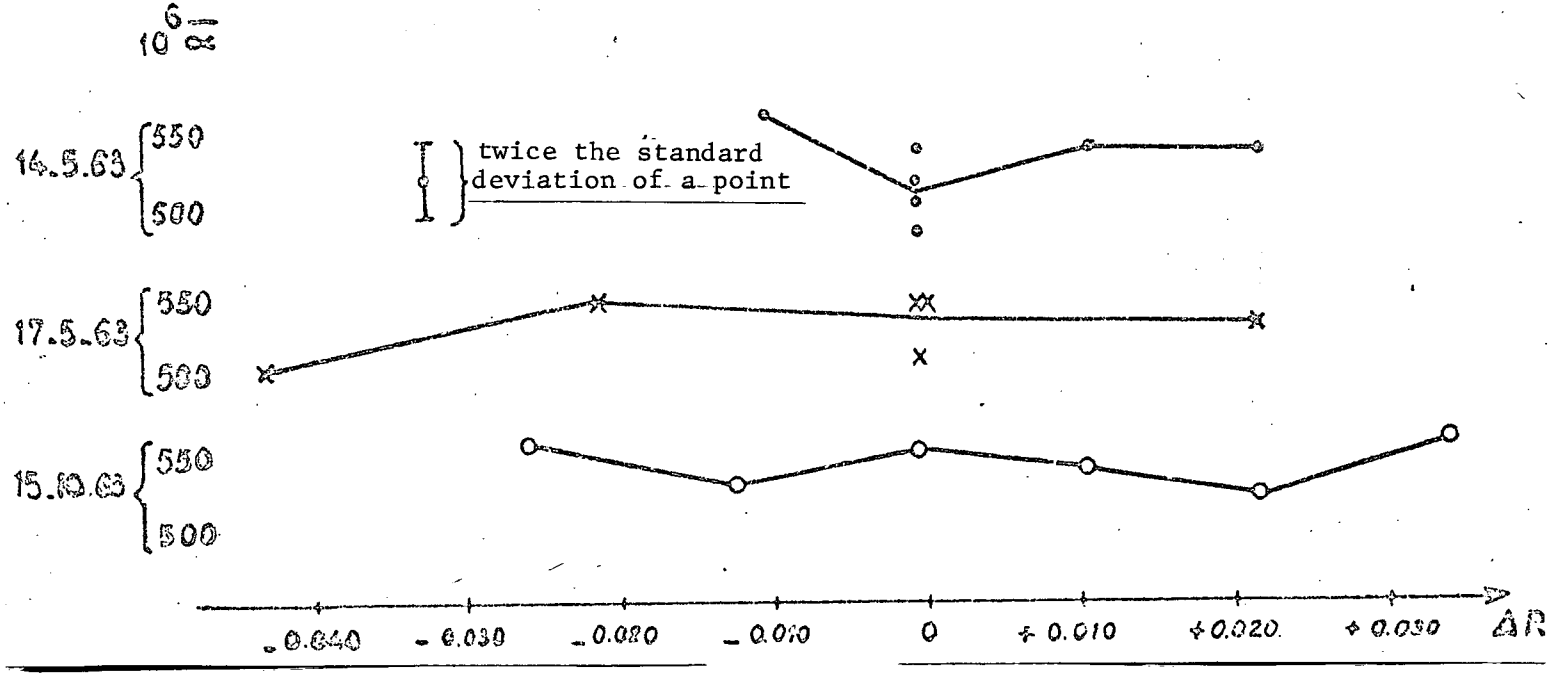

Figure 6. Variation of the distortion coefficient as a function of the voltage ratio ΔR. The value of the standard deviation $\bar{\alpha}$ is indicated.

The standard deviation of a $\bar{\alpha}_i$ is on the order of 0.018. The distortion therefore varies significantly with direction. Figure 7 shows this. It also varies from one experiment to the next, even if no disassembly takes place. In any case, let us remember the value of $\bar{\alpha}_x$ of October 15, 1963

$$10^3\bar{\alpha}_x = 0{,}519 \pm 0{,}018$$

It is compatible with the one derived from the interference rings for the group of plates taken in October, 1963:

$$10^3\alpha = 0{,}527 \pm 0{,}008.$$

We also investigated the influence of the points made along the edges of the plate. The shutter limits the image to a band, which is elongated along the x axis and has a width of about 9 mm. The image of the photo cathode has a diameter of about 14 mm. The results of Table III were obtained by always stopping the measurements 4.5 mm from the center. A second solution was determined by extending the measurement as far as possible along the x-axis. Table IV allows a comparison of the results.

The general average of α_x is:

for entire range of x: $\alpha_x = 0.505\ 10^{-3} \pm 0.004\ 10^{-3}$

for x limited to 4.5 mm: α_x limited $= 0.487\ 10^{-3} \pm 0.011\ 10^{-3}$

The standard deviations were calculated from the dispersion of the series having the same date. The difference in the two values of α_x is very significant. It is mostly due to two abnormal values of α for x limited on May 14, 1963, Plate No. 7, and October 15, 1963, Plate No. 7. The two values correspond to voltages which are quite different from optimum voltages. If

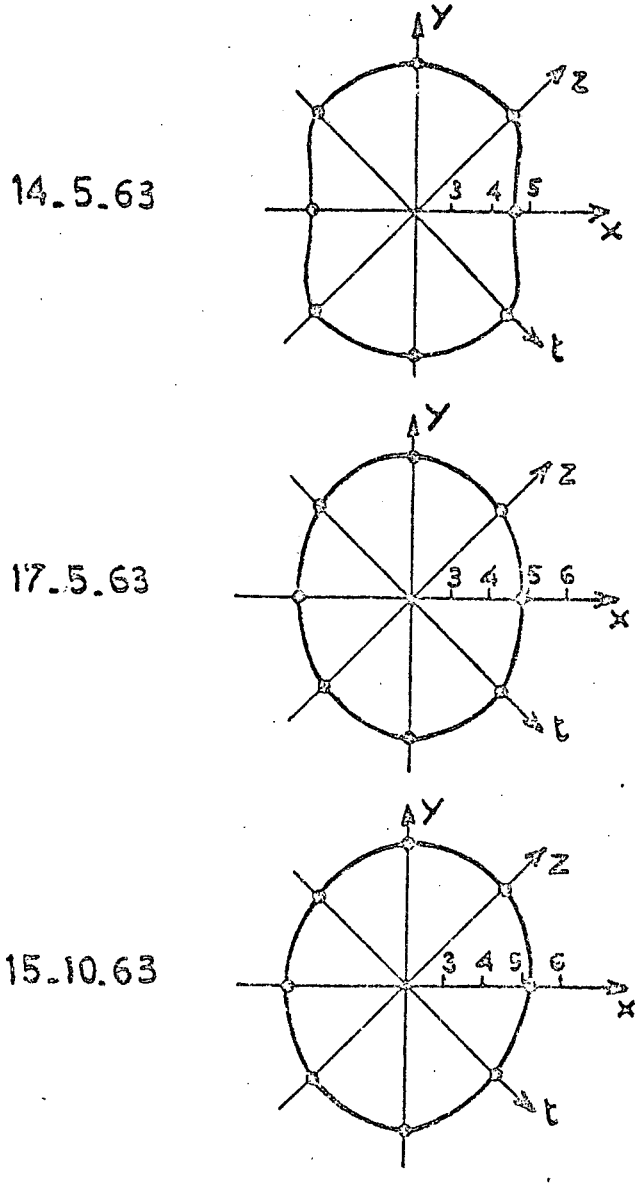

Figure 7. Variation of the distortion coefficient as a function of direction. Ox is measured in the height direction of the shutter. The unit of α is 10^{-4}.

it were desired to compensate for this small difference by using a fifth order distance term, it would be necessary to give a positive value to this term. This sign would be opposite to that given by G. Wlérick and his associates [4]. At 4.5 mm from the center, the deviation of the distortion correction is given as follows, according to which value of α is used:

$$(0{,}505 - 0{,}487)10^{-3} \times (4{,}5)^3 = 0{,}0016\,\text{mm}$$

This deviation is largely compensated by the variation in k. On the average we have, /24

for x full: k full = 0.80146

for x limited: k limited = 0.80172

If we write the following expression reckoning X and x from C:

$$X = kx(1 + \beta x^2) = kx(1 + \alpha k^2 x^2),$$

we get λ by differentiating with respect to k and α, and ignoring the term containing αdk:

$$dX = x\,dk + k^3 x^3\,d\alpha.$$

For X = 4.5 mm, i.e., $x = \frac{1}{k} \times 4{,}5 = 5{,}6$ in microns, we have:

$$dX = -1{,}5 + 1{,}6 = +0{,}1.$$

The difference between the two representations does not reach 1 μ at any point. It is therefore not useful to use the fifth order term.

We should also note that by taking the entire length for x, we arrive at the following result for May 14, 1963

$$\bar{\alpha}_x = 0{,}487\ 10^{-3} \quad \text{instead of} \quad \bar{\alpha}_x = 0{,}453\ 10^{-3}.$$

The anomaly in the distribution of α as a function of direction disappears and no further variation in the distortion is observed for experiments in which there was neither disassembly nor shifting of the electronic optics.

TABLE IV. COMPARISON OF THE RESULTS ALONG THE x DIRECTION, FOR MEASUREMENTS EXTENDED TO THE EDGE (x FULL RANGE) OR TERMINATED AT 4.5 mm FROM THE CENTER (x LIMITED).

Date and No. of the Plate	Full		Limited		Full		Limited	
	k	σ_k	k	σ_k	$10^3\alpha$	$10^3\sigma_\alpha$	$10^3\alpha$	$10^3\sigma_\alpha$
13. 2.63	0,8230	0,0017	0,8227	0,0018	0,453	0,032	0,461	0,058
14. 5.63								
4	0,7976	0,0011	0,7981	0,0013	0,484	0,025	0,468	0,058
5	7982	11	7983	09	470	28	453	42
6	7997	16	8007	16	488	37	408	69
7	8018	13	8033	18	463	30	362	79
10	7991	11	7992	13	534	27	495	56
11	7996	10	7999	10	497	23	471	44
12	8002	13	7999	13	476	30	512	58

TABLE IV (Continued)

Date and No. of the Plate	Full		Limited		Full		Limited	
	k	σ_k	k	σ_k	$10^3\alpha$	$10^3\sigma_\alpha$	$10^3\alpha$	$10^3\sigma_\alpha$
17. 5.63								
5	0,7971	0,0013	0,7975	0,0012	0,514	0,031	0,502	0,053
6	8022	09	8024	10	496	22	490	45
7	8058	08	8055	09	484	20	510	40
10	7991	13	7994	12	498	31	480	55
11	7988	09	7986	09	483	22	518	41
12	7999	14	8004	10	489	34	466	48
13.10.63								
	0,8040	0,0022	0,8041	0,0011	0,515	0,054	0,506	0,051
15.10.63								
7	0,7979	0,0018	0,7990	0,0021	0,544	0,042	0,443	0,104
8	7992	15	7991	14	532	34	535	0,069
9	7995	17	7997	13	543	40	537	64
10	8012	14	8008	18	541	31	577	84
11	8026	13	8033	16	555	31	497	76
12	8041	14	8043	15	539	31	527	71

c - Coordinates of the Distortion Center /25

These coordinates do not exhibit a systematic variation with the voltages applied to the electrodes. For the three series of 7 and 6 plates, on the average we find the following result, where the origin is taken at the center of the image of the photo cathode:

14. 5.63	$x = +\ 0{,}26 \pm 0{,}02$ mm $y = +\ 0{,}58 \pm 0{,}06$ $z = +\ 0{,}61 \pm 0{,}07$ $t = -\ 0{,}04 \pm 0{,}09$
17. 5.63	$x = +\ 0{,}15 \pm 0{,}03$ $y = +\ 0{,}43 \pm 0{,}05$ $z = +\ 0{,}49 \pm 0{,}06$ $t = -\ 0{,}11 \pm 0{,}07$
15.10.63	$x = -\ 0{,}26 \pm 0{,}03$ $y = +\ 0{,}12 \pm 0{,}02$ $z = +\ 0{,}05 \pm 0{,}06$ $t = -\ 0{,}20 \pm 0{,}03$

Each of these coordinates results in a geometric locus of C and is shown in Figure 8. The solution using the method of least squares is the following, taking into account the standard deviation of each of these coordinates

Date	in mm		deviations (obs-calc), in mm			
	x	y	x	y	z	t
14. 5.63	+0,27	+0,55	−0.01	+0,03	+0,04	−0,16
17. 5.63	+0,17	+0,43	−0,02	0,00	+0,07	+0,07
15.10.63	−0,21	+0,12	−0,05	0,00	+0,11	+0,03

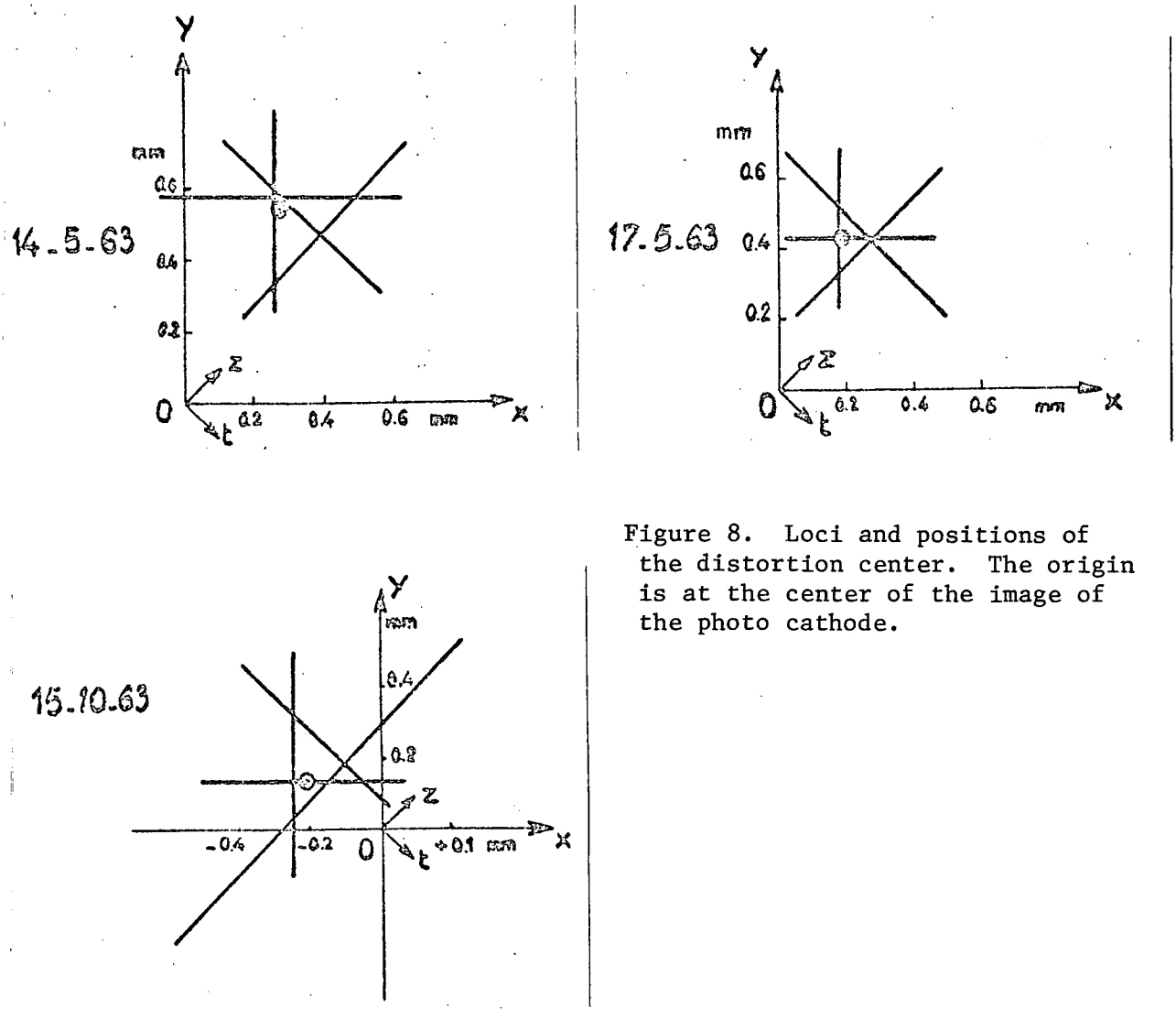

Figure 8. Loci and positions of the distortion center. The origin is at the center of the image of the photo cathode.

In reality, the coordinates and the standard deviations given above refer to a point of the photocathode which may be offset from its center by several hundredths of a millimeter. The center of distortion did not seem to be displaced from one plate series to another, as long as the optical electronics was not disassembled and the camera was not moved. On the other hand, we are certain that disassembly and displacements made it vary.

CHAPTER III

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CONCLUSIONS

The present work is based on the study of 54 plates taken with an electronic camera. They were measured in several ways, so that 137 independent values of the distortion coefficient were obtained.

Deformations of the Image

The optical image is deformed.

It is enlarged by an amount which varies linearly with the ratio of voltages applied to the electrodes of the camera.

The other deformations do not depend on the voltage.

There is affinity of the image due (to a great extent) to the deformations of its base.

There exists a center of distortion. The distances D reckoned on the image from this center must be corrected by truncating the term αD^3. α varies slightly (10%) with direction. The order of magnitude of the correction is 50 μ for d = 4.5 mm.

The form and size of the aberrations in the course of the experiments carried out at the same location are retained, if there is no disassembly of the electronic optics.

Practical Conclusions

The experiments which we carried out showed that the deformations of the optical image introduced by the electronic camera are quite considerable, but that they are faithfully reproduced on all plates which can be taken without opening the tube. For these deformations we gave a geometrical description, which is sufficient to make the electronic image identical with the optical image to an approximation of one micron, using the correct numerical coefficients.

When we perform measurements of the plates, each case must be examined individually according to the required accuracy and the properties of the object being observed.

If an accuracy of 1 micron is desired for the two coordinates over the entire plate, the simplest thing is to superimpose on the image of the sky an image of a grid. Then the coordinates are measured with respect to the lines of this grid. If this is not possible, then the plate to be studied must be framed by two grid plates, and the values of the seven parameters allowing the reconstruction of the image must be determined, as we have done. These seven parameters are:

- the two coordinates of the distortion center,
- the two extreme values of the distortion coefficient,
- the direction of the greatest distortion (assuming an eliptical distribution),
- the affinity coefficient,
- the affinity direction.

In this case, it is better not to use emulsions on a plastic base because their own deformations are very far from being negligible.

If accurate measurements are carried out only in one direction and along a straight band passing through the center of the plate (case of a slightly enlarged spectrum), the affinity can be ignored and the following three parameters will suffice:

- abscissa of the distortion center in this direction,
- distortion coefficient,
- enlargement.

The correction by calculation then becomes simplest of all. The values of the parameters can be derived from one grid image.

Finally, often an accuracy of 5 μ is sufficient. In this case, the deviation between the center of distortion and the image on the center of the photo cathode can be ignored, as well as the anisotropy of the distortion. The correction is then carried out in a very simple way by calculation. Often the value of the distortion coefficient can be derived from the image itself. If this is not possible, a check of this value can be made by saving one plate for the grid plate in each experiment (i.e., for the series of plates of a magazine).

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